

# Precise Measurements of the $Q$ Factor of Dielectric Resonators in the Transmission Mode—Accounting for Noise, Crosstalk, Delay of Uncalibrated Lines, Coupling Loss, and Coupling Reactance

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**Abstract**—Accurate measurements of the unloaded  $Q_o$  factor of microwave resonators are necessary in many microwave applications. The most accurate values of  $Q_o$  can be obtained by  $Q$ -circle fits from multifrequency  $S$ -parameter data. Practical measurement systems cause  $S$ -parameters of the resonators to be distorted from the circular ideal shape, rotated, and shifted from the origin resulting in errors in the  $Q$ -factor values. A novel  $Q$ -factor measurement method has been developed based on equations derived for resonators working in the transmission mode and fractional linear circle-fitting techniques. The transmission-mode  $Q$ -factor (TMQF) technique removes measurement effects of noise, noncalibrated measurement cables, connectors, coupling structures, crosstalk between the coupling loops, and impedance mismatch from the measurement data. The TMQF is especially useful in cryogenic measurements of high-temperature-superconductor thin films and dielectrics since these measurements are typically done in the transmission mode and contain cables and connectors that are difficult to calibrate. The accuracy of the TMQF is better than 1% for practical measurement ranges and the method is applicable to a wide range of coupling. The range of  $Q$  factors measurable is from  $10^3$  up to  $10^7$ .

**Index Terms**—Dielectric resonators, measurements effects,  $S$ -parameters, transmission-mode measurements, TMQF, unloaded  $Q$  factor.

## I. INTRODUCTION

M EASUREMENTS of the unloaded  $Q_o$  factor play an important role in the assessment of performance of systems and material properties. The  $Q$ -factor values are especially useful in measurements of selectivity of systems, as well as surface resistance of conducting and superconducting materials and complex permittivity of dielectric materials [1]–[3].

By definition, the quality factor is a ratio of the maximum energy  $W_{\max}$  stored within a system to the energy dissipated  $W_d$  by this system [4] as follows:

$$Q = \frac{2\pi W_{\max}}{W_d}.$$

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The unloaded  $Q_o$  factor of the system is obtained from the measured loaded  $Q_L$  factor of this system together with external circuitry and coupling coefficients. For resonators in the transmission mode  $Q_o = Q_L(1 + \beta_1 + \beta_2)$ , where the coupling coefficients  $\beta_1$  and  $\beta_2$  are defined as a ratio of the external power dissipated to power dissipated in the system. For resonators working in the reflection mode  $Q_o = Q_L(1 + \beta)$ .

Under a very weak coupling condition, the unloaded  $Q_o$  factor can be approximated by the loaded  $Q_L$  factor ( $Q_o \approx Q_L$ ), eliminating the need for  $\beta$  measurements. However, measurements with low coupling usually result in low SNR ratio that may affect the accuracy of the measurements. Hence, it is desirable to use sufficiently high levels of coupling.

Methods used to obtain the loaded  $Q_L$  factor and  $\beta$  are based on measurements of various types of system response: impedance [5], [6], power [5], [7]–[10], voltage standing-wave ratio (VSWR) [5] and  $S$ -parameters [4], [11]–[15]. Measurements of a chosen response are done either in the frequency or time domains. Although the 3-dB “three frequencies” method is used often to determine  $Q_L$ , the most accurate results are obtained by fitting a circle to multifrequency data. This is a more accurate method than the 3-dB method because all points around the resonance contribute to the definition of the circle. While the multifrequency tests were very time consuming when first proposed by Gintzon [5], with the availability of vector network analyzers, they should become a standard practice for accurate measurements.

Most of the  $Q$ -factor measurement techniques have been developed based on ideal models of resonator systems. Hence, any discrepancy between the ideal resonator and practical resonator measurement system and inadequacies of calibration procedures result in errors in measured  $Q$ -factor values, as illustrated in Section II. Recently, there has been strong interest in precise microwave characterization of new (and some “old”) materials at cryogenic temperatures for applications to wireless communications. Most cryogenic measurement systems contain cables and adaptors, which cannot easily (if ever) be calibrated. Hence, a new and accurate  $Q$ -factor measurement method was needed, which accounts for parasitic measurement effects (noise, noncalibrated cables, connectors, coupling structures, crosstalk, and impedance mismatch) and this is the goal of this paper.

An existing method [4] accounts for some of the mentioned effects, but is applicable to the reflection mode resonators only.

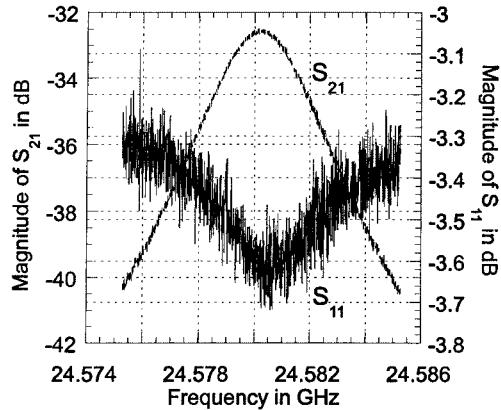


Fig. 1. Magnitude traces of  $S_{21}$  and  $S_{11}$  for a resonator with  $f_{res} = 24.58$  GHz,  $Q_L = 5564$ ,  $S_{21\max} = -32.6$  dB, BW = 4.42 MHz, and span = 10 MHz.

As mentioned earlier, most of microwave characterization of materials is usually done in the transmission mode due to a much higher SNR ratio.

## II. REVIEW OF EFFECTS INTRODUCED BY THE MEASUREMENT SYSTEM ON THE DIELECTRIC-RESONATOR RESPONSE

The development of an accurate curve-fitting technique to obtain the unloaded  $Q_o$  factor of microwave resonators requires a good understanding of practical effects introduced by real measurement systems. The  $S$ -parameters of an ideal resonator measured near the resonance should form circles in the complex plane centered at the origin. However, due to effects associated with external measurement circuitry, the measured responses of a dielectric resonator are distorted from their ideal circular shape and often do not pass through the origin of the complex plane. Phenomena that may affect the  $S$ -parameters of dielectric resonators are: 1) noise; 2) electrical delay introduced by transmission lines; 3) losses due to transmission lines; 4) connectors; 5) coupling structures; 6) impedance mismatch; and 6) crosstalk between the coupling loops of transmission-mode resonators. Identified phenomena [16] are documented and discussed below.

### A. Effects of Noise

Noise is always present in any measurement data as random variations of amplitude and phase caused by thermal effects, instabilities in microwave sources, and vibrations of a resonator. Under weak coupling conditions, the noise in a reflection trace is usually significantly higher than in the transmission trace. Fig. 1 shows the  $S_{21}$ - and  $S_{11}$ -parameters measured using the same coupling ( $S_{21\max} = -32.6$  dB) on the input and output ports for a Hakki–Coleman sapphire resonator in a copper cavity resonating at 24.58 GHz at room temperature with the loaded  $Q_L$  factor of 5564. The  $S_{21}$  magnitude trace averaged over 16 sweeps is well defined, but the  $S_{11}$  magnitude trace is clearly much more noisy.

When curve-fitting procedures are applied to raw measurement data, the effect of noise can be eliminated to a large extent. However, a low SNR ratio can affect the accuracy of the fitting

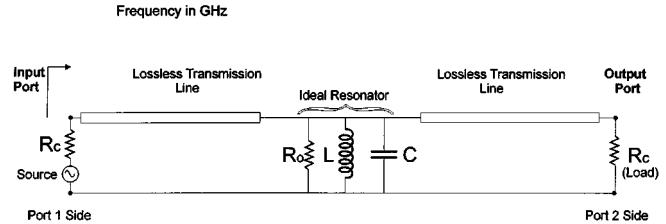


Fig. 2. Circuit model of a measurement system based on a dielectric resonator.

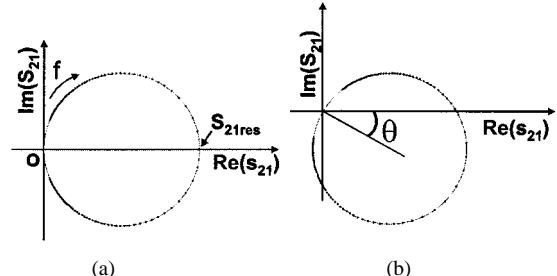


Fig. 3.  $Q$  circle. (a) Ideal. (b) With delay due to transmission lines.

techniques, especially when the amount of noise exceeds a certain limit [4]. This is why transmission-mode measurements are often used instead of the reflection mode, as the SNR is usually high enough for curve-fitting techniques to be applied accurately.

### B. Effects of Electrical Delay Introduced by Uncalibrated Transmission Lines on the Measured Characteristics of Dielectric Resonators

Consider a circuit model of an ideal transmission-mode resonator modeled by parallel  $RLC$  elements together with lossless transmission lines connecting the resonator to a microwave source and a load, as shown in Fig. 2. Uncalibrated transmission lines introduce a frequency-dependent electrical delay, equivalent to a frequency-dependent phase shift [4],  $\phi(f) = \phi_{ref}f/f_{ref}$ , where  $\phi_{ref}$  is the phase introduced by the transmission line at the arbitrary reference frequency  $f_{ref}$ . If the length of the transmission lines in the measurement system is zero, the  $S_{21}Q$  circle of the system passes through the origin and is centered on the real axis, as shown schematically in Fig. 3(a). The detuned point, which represents a point at a frequency far away from the resonance, lies at the origin in this case. For any nonzero length of the transmission lines, an electrical delay is introduced and causes all the points on the ideal  $Q$  circle to be rotated about the origin. The rotation angle  $\theta$  depends on the length of transmission line [see Fig. 3(b)].

Rotation of each point on the  $Q$  circle in Fig. 3 is frequency dependent. This phenomenon is illustrated in Fig. 4 showing the phase of the  $S_{11}$ -parameter changing linearly due to frequency dependence of the delay introduced by transmission lines. Measurements shown in Fig. 4 were made with a 25-GHz sapphire resonator in the copper cavity at room temperature exhibiting  $Q_L$  of 2574 for  $S_{21\max}$  of  $-8.1$  dB. The irregularity at the mid-span is due to phase characteristics of the microwave resonator at the resonance.

The frequency dependence of the phase shift causes larger rotation of points on the  $Q$  circle at higher frequencies in the

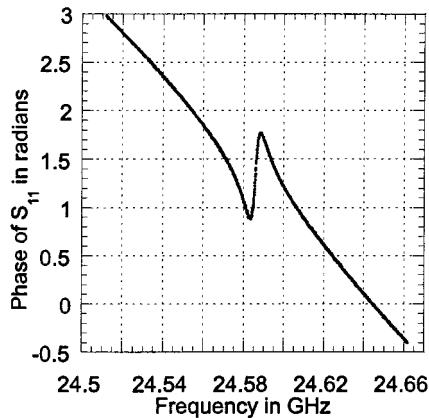


Fig. 4. Phase of  $S_{11}$  trace versus frequency for the resonator with  $Q_L = 2574$ , uncalibrated cables of 40 cm per cable,  $f_{res} = 24.59$  GHz, span = 150 MHz, and BW = 4.42 MHz.

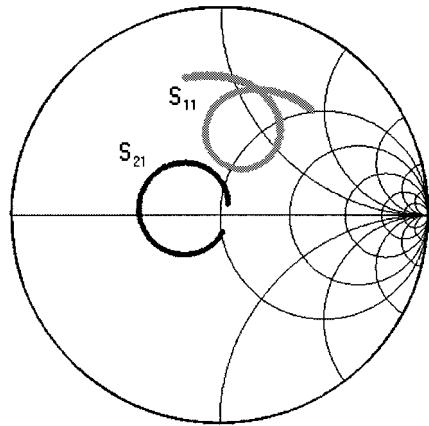


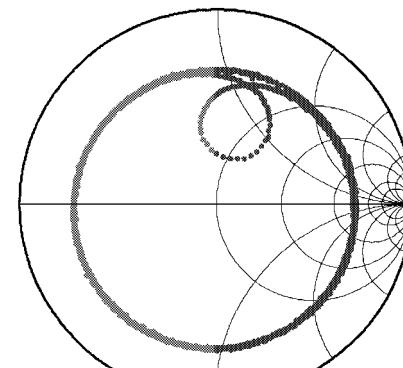
Fig. 5. Measured “balloon”-shaped reflection  $S_{11}$   $Q$  circle and the transmission  $S_{21}$   $Q$  circle for the resonator with  $Q_L = 2574$ ,  $S_{21\max} = -8.1$  dB,  $f_{res} = 24.59$  GHz, BW = 9.55 MHz, and span = 50 MHz, and cable length of 40 cm on each port.

complex frequency plane than the rotation of points at lower frequencies. As a result, a distortion of the circular shape of the  $Q$  circle to a “balloon” shape occurs. This phenomenon is illustrated in Fig. 5 for the  $S_{11}$   $Q$  circle of the dielectric resonator with  $Q_L$  of 2574 and  $S_{21\max}$  of  $-8.1$  dB. The transmission  $S_{21}$   $Q$  circle measured for the same case (also shown in Fig. 5) is significantly less distorted.

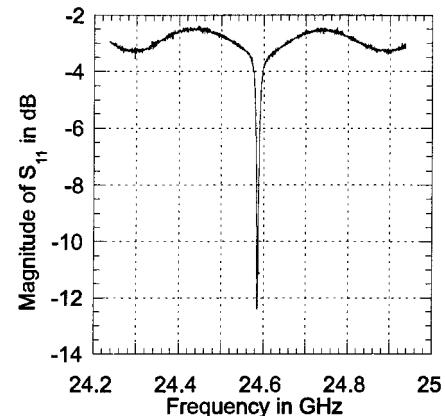
Cables connecting a dielectric resonator to the measurement system also introduce attenuation of microwave signals. The effect of attenuation on a reflection ( $S_{11}$  or  $S_{22}$ )  $Q$  circle is a reduction in the diameter, as well as a movement away from the unit circle on the Smith Chart. Effects of loss introduced by coupling and system elements are discussed in Section II-D.

### C. Effects of Impedance Mismatch

Impedance mismatch between sections of a measurement system causes reflections of microwave signals at mismatched points. For a matched system, the center of the circular  $S_{11}$  or  $S_{22}$  trace observed in off-resonance regions (for frequencies far from the resonance) is expected to lie at the origin of the Smith Chart. However, we observed a noticeable offset of the center of the off-resonance traces of  $S_{11}$  and  $S_{22}$  from the origin in



(a)



(b)

Fig. 6. Observed mismatch effect in the  $S_{11}$ . (a) Resonance  $Q$  circle and the off-resonance circle. (b) Magnitude trace for the resonator with  $Q_L = 2574$ ,  $S_{21\max} = -8.1$  dB,  $f_{res} = 24.59$  GHz, and span = 400 MHz.

the wide-band measurements. Fig. 6(a) presents a mismatch effect observed in the off-resonance part of the reflection  $S_{11}$  trace of the 25-GHz sapphire resonator in the copper cavity at room temperature. The  $S_{11}$   $Q$  circle (which is evidently distorted due to transmission-line phase effects, as discussed in Section II-B) is attached to the larger off-resonance circle. The center of the off-resonance circle clearly does not coincide with the center of the Smith Circle, indicating the presence of impedance mismatch.

The effect of impedance mismatch can be observed clearly in the magnitude trace of reflection response. If mismatch is not present, then the magnitude of the response in the off-resonance region is at a constant level. In the presence of mismatch, the magnitude of  $S_{11}$  changes periodically with frequency, except at the resonance peak, as shown in Fig. 6(b).

The offset of the center of the off-resonance trace from the origin due to mismatch also causes a nonlinear effect in the phase of reflection response  $S_{11}$  and  $S_{22}$ . If there were no mismatch and, therefore, no offset of the center of the off-resonance circle from the origin, then a periodic linear phase response is expected, with the phase of each linear section going from  $+\pi$  to  $-\pi$  rad. With the mismatch present, the phase response deviates from a straight line. This is illustrated in Fig. 7, showing the mismatch effect on the phase of  $S_{11}$  for the resonator under the same conditions as in Fig. 6(a). The small irregularity in the phase at mid-span is associated with the resonance.

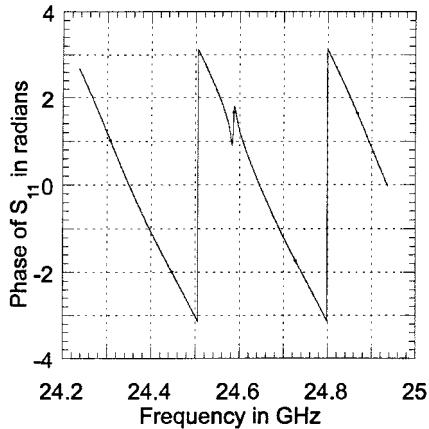


Fig. 7. Phase of the  $S_{11}$  response for the resonator with  $Q_L = 2574$ ,  $S_{21\max} = -8.1$  dB,  $f_{res} = 24.59$  GHz, and span = 400 MHz.

The effect of the impedance mismatch is considered important because it can alter significantly both the magnitude and phase of the reflection  $S_{11}$  or  $S_{22}$  responses around the resonance. While this effect has not yet been investigated thoroughly, the frequency dependence of mismatches can be observed experimentally using a time-domain reflectometer.

#### D. Effects of Coupling Loss and Reactance on the Measured Characteristics of Dielectric Resonators

Coupling of a resonator to an external circuit usually alters measured responses of the resonator. Coupling structures are usually in the form of a coupling loop (or iris) and have finite resistances that contribute to the microwave losses in the system. Also, microwave energy is stored in the coupling structure and this phenomenon is usually modeled as a reactance that is frequency dependent. However, for high- $Q$ -factor systems (when the bandwidth of measurements is very small), it may be justifiable to assume that the reactance is constant in the frequency band around the vicinity of the resonance [4].

Our measurements using Hakki–Coleman sapphire resonators in the copper cavity have shown that the frequency dependence of coupling reactances did not influence the magnitude of the off-resonance reflection traces. If the frequency dependence of the reactance were significant, then the reflection vector would have not followed the path of a perfect circle. The off-resonance  $S_{11}$  trace of Fig. 6(a) is clearly circular, indicating that the assumption of a constant coupling reactance is justified for the resonator with a loaded  $Q_L$  factor of about 2600. The assumption of the constant coupling becomes more accurate for higher  $Q_L$  factors, hence, there is no need to consider frequency dependence of coupling reactances for resonators with  $Q$  factors higher than 1000.

Coupling losses and coupling reactance effectively reduce the coupling to the resonator and decrease the diameter of  $S_{21}$   $Q$  circles. The effect of coupling loss (and any other loss in the system) can be investigated by examining the location of the reflection mode ( $S_{11}$  or  $S_{22}$ )  $Q$  circles on the Smith chart complex plane. Fig. 8 shows simulated  $Q$  circles varying in size and position for different coupling loss.

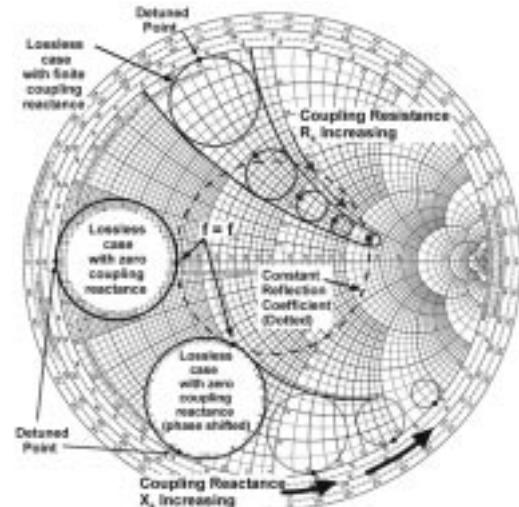


Fig. 8. Reflection  $Q$  circles for various cases of coupling loss and coupling reactance.

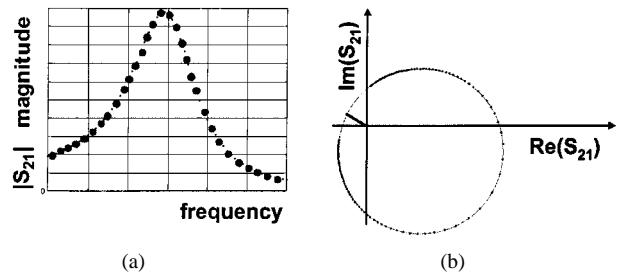


Fig. 9. Effect on crosstalk on  $S_{21}$  parameter. (a) Magnitude. (b)  $Q$  circle.

For a system with no coupling losses, but with a transmission-line delay, the reflection circle has the center on the real axis and the detuned point at  $\Gamma = -1$  of the Smith circle. When coupling losses are present, the reflection  $Q$  circle no longer touches the circumference of the Smith circle. When the coupling reactance is held constant and the coupling losses are increased from zero, the  $Q$  circle moves away from the perimeter of the Smith circle with decreasing diameter, while all points on the circle maintain the same reactance. When the coupling losses are held constant and the coupling reactance is increased, the  $Q$  circle moves with decreasing diameter in the direction of the change in reactance with points moving along lines of constant resistance, as shown in Fig. 8.

#### E. Effects of Crosstalk

In the presence of crosstalk between the two ports of a resonator, the magnitude transmission response deviates from the perfect Lorentzian “bell shape,” as illustrated in Fig. 9(a). However, the crosstalk does not distort the shape of the  $Q$  circles.

If no crosstalk is present, then the detuned point of the  $S_{21}$  circle lies at the origin. When crosstalk is present, the detuned point moves away from the origin [see Fig. 9(b)]. In the measurements shown in Fig. 5(b), the effect of crosstalk is clearly evident because a full circle fitted to the  $S_{21}$  trace would not pass through the origin.

TABLE I  
POSSIBLE WAYS TO ACCOUNT FOR DISTORTION EFFECTS ON  $S$ -PARAMETERS OF DIELECTRIC RESONATORS

PARASITIC EFFECT	ACCOUNTING PROCEDURE
Noise (amplitude and phase)	Curve fitting
Coupling loss and reactance	Incorporation into a model
Transmission line loss and connector loss	Incorporation into a model
Crosstalk	Circle offset in curve fitting or incorporating into a model
Frequency dependant delay due to transmission lines	Calibration or a phase correction procedure

*F. Summary—How to Deal With Effects Introduced by Real Measurement Systems on Characteristics of Dielectric Resonators*

Measured  $S$ -parameters of dielectric resonators are distorted by noise, electrical delay introduced by transmission lines, losses due to uncalibrated transmission lines and coupling structures, impedance mismatch, and crosstalk. Some of the effects can be eliminated through a calibration procedure. Other distortions need to be removed from measured raw data through fitting procedures to obtain more accurate values of  $S$ -parameters and, thus, the loaded and unloaded  $Q$  factor of the resonator. A summary of possible ways of removing various distortions is presented in Table I.

**III. TRANSMISSION-MODE  $Q$ -FACTOR TECHNIQUE FOR ACCURATE DETERMINATION OF THE UNLOADED  $Q_o$  FACTOR OF MICROWAVE RESONATORS BASED ON MEASUREMENTS OF  $S$ -PARAMETERS**

The following fundamental assumptions have been made to develop a feasible, accurate, and useful method to determine the  $Q_o$  factor of dielectric resonators.

- 1) The method is to be applicable to resonators working in the transmission mode.
- 2) The full equation to calculate the unloaded  $Q_o$  factor is to be used, namely,

$$Q_o = Q_L (1 + \beta_1 + \beta_2) \quad (1)$$

implying the following.

- a) A new equation is needed to relate the loaded  $Q_L$  factor with  $S_{21}$ -parameters around the resonance through a circuit model of the dielectric resonator system.
- b) A new equation is needed to relate coupling coefficients  $\beta_1$  and  $\beta_2$  with  $S_{11}$  and  $S_{22}$  reflection parameters for transmission-mode resonators.
- 3) Derivation of equations of the loaded  $Q_L$  factor, coupling coefficient, and frequency versus the  $S_{21}$  relationship is to be based on an accurate model of a real measurement system as feasible.
- 4) Important parasitic effects of a real resonator measurement system described in Section II, but not included in the circuit model of the system, are to be accounted for in the circle-fitting procedure applied to measured  $S$ -parameter responses of the system.

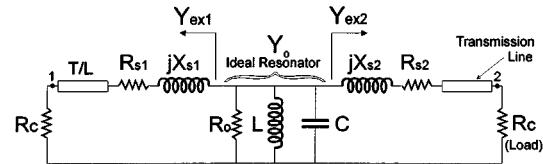


Fig. 10. Circuit model of a transmission resonator system.

To derive equations relating the  $S$ -parameters with the loaded  $Q_L$  factor, coupling coefficients, and the resonant frequency, the circuit model of a transmission-mode resonator illustrated in Fig. 10 has been used.

The three components  $R_o$ ,  $L$ , and  $C$  of shunt admittance  $Y_o$  represent the ideal dielectric resonator. The shunt elements determine the unloaded  $Q_o$  factor of the system that can be described using a narrow-band approximation [1] as

$$Y_o = \frac{1}{R_o} \left\{ 1 + j2Q_o \frac{(\omega - \omega_o)}{\omega_o} \right\} \quad (2)$$

where  $Q_o$  is the unloaded quality factor of the resonator and the radian resonant frequency  $\omega_o$  of the ideal resonator given by

$$\omega_o = \frac{1}{\sqrt{LC}}. \quad (3)$$

The admittances  $Y_{\text{ex}1}$  and  $Y_{\text{ex}2}$  denote external admittances seen on each port of the ideal resonator in Fig. 10 and are given by

$$y_{\text{exp}} = \frac{1}{R_{sp} + R_c + jX_{sp}} = G_{\text{exp}} + jB_{\text{exp}} \quad (4)$$

where  $p$  of 1 or 2 denotes the input and output port of the resonator respectively. Resistances  $R_{s1}$  and  $R_{s2}$  represent losses of coupling structures, connectors, and losses of transmission lines connected to two ports of the resonator; reactances of the coupling structures are modeled by  $X_{s1}$  and  $X_{s2}$ . The resistance  $R_c$  represents an internal resistance of a microwave source, a load resistance and characteristic impedance of transmission lines at both ports.

The circuit diagram of Fig. 10 can be considered as a cascaded connection of three networks and can be described using the  $ABCD$  matrix as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R_{s1} + jX_{s1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_o & 1 \end{bmatrix} \begin{bmatrix} 1 & R_{s2} + jX_{s2} \\ 0 & 1 \end{bmatrix}. \quad (5)$$

Transmission lines have been treated as ideal and have not been included in (5). Their losses and delay are accounted for later.

The  $S$ -parameters of the total resonator system can be obtained from the  $ABCD$ -parameters using equations after [2], namely,

$$S_{21} = \frac{2R_c}{AR_c + B + CR_c^2 + DR_c} \quad (6)$$

$$S_{11} = \frac{(A - D)R_c + B - CR_c^2}{(A + D)R_c + B + CR_c^2} \quad (7)$$

$$S_{22} = \frac{(D - A)R_c + B - CR_c^2}{(A + D)R_c + B + CR_c^2}. \quad (8)$$

#### A. Derivation of Relationships Between $Q_L$ and $S_{21}$ (and $S_{11}$ and $S_{22}$ )

Substituting (5) into (6), the following expression for the transmission coefficient  $S_{21}$  can be obtained as follows:

$$S_{21} = \frac{2R_c Y_{\text{ex1}} Y_{\text{ex2}}}{Y_{\text{ex1}} + Y_{\text{ex2}} + Y_o}. \quad (9)$$

The denominator of (9) is equal to the total admittance  $Y_L$ , where

$$\begin{aligned} Y_L &= Y_o + Y_{\text{ex1}} + Y_{\text{ex2}} \\ &= G_o + G_{\text{ex1}} + G_{\text{ex2}} + j2Q_o G_o \frac{\omega - \omega_o}{\omega_o} \\ &\quad + j(B_{\text{ex1}} + B_{\text{ex2}}). \end{aligned} \quad (10)$$

The resonant frequency  $\omega_L^1$  of the loaded resonator can be found from (10) assuming the imaginary part of the total admittance  $Y_L$  to be zero, namely,

$$\omega_L = \omega_o \left[ 1 - \frac{B_{\text{ex1}} + B_{\text{ex2}}}{2Q_o G_o} \right]. \quad (11)$$

On the basis of (9)–(11), the frequency dependence near the resonance of the complex transmission coefficient  $S_{21}$  for transmission-mode resonators can be expressed by a novel equation relating  $S_{21}$  with the loaded  $Q_L$  factor, coupling coefficients, and frequency

$$\begin{aligned} S_{21}(\omega) &= \frac{2R_c Y_{\text{ex1}} Y_{\text{ex2}}}{G_o (1 + \beta_1 + \beta_2) \left[ 1 + j2Q_L \frac{(\omega - \omega_L)}{\omega_o} \right]} \\ &\approx \frac{2R_c Y_{\text{ex1}} Y_{\text{ex2}}}{G_o (1 + \beta_1 + \beta_2) \left[ 1 + j2Q_L \frac{(\omega - \omega_L)}{\omega_L} \right]} \end{aligned} \quad (12)$$

where  $\beta_1$  and  $\beta_2$  are the coupling coefficients of the loaded resonator

$$\beta_1 = \frac{G_{\text{ex1}}}{G_o} \text{ and } \beta_2 = \frac{G_{\text{ex2}}}{G_o} \quad (13)$$

and  $Q_L$  is the  $Q$  factor of the loaded resonator

$$Q_L = Q_o \left( \frac{G_o}{G_o + G_{\text{ex1}} + G_{\text{ex2}}} \right). \quad (14)$$

<sup>1</sup>The loaded  $\omega_L$  and unloaded  $\omega_o$  resonant frequencies of the dielectric resonator have a similar meaning as the loaded and unloaded  $Q$  factors.

The approximation of  $\omega_o \cong \omega_L$  can be considered acceptable for systems with high- $Q$  factors. Equation (12) has a fractional linear form [4], sometimes called a bilinear form

$$S_{21} = \frac{a_1 t + a_2}{a_3 t + 1} \quad (15)$$

where the complex constants  $a_1$ ,  $a_2$ , and  $a_3$  are

$$a_1 = 0 \quad a_2 = \frac{2R_c Y_{\text{ex1}} Y_{\text{ex2}}}{G_o (1 + \beta_1 + \beta_2)} \quad a_3 = jQ_L \quad (16)$$

and “ $t$ ” is a normalized frequency variable defined by

$$t = 2 \left( \frac{\omega - \omega_L}{\omega_L} \right). \quad (17)$$

The parameter  $a_3$ , the imaginary part of which is equal to the loaded  $Q_L$  factor, namely,

$$Q_L = \text{Im}[a_3] \quad (18)$$

is obtained using a fractional linear curve-fitting procedure applied to (12). For completeness, the relationships between the reflection parameters  $S_{11}$  and  $S_{22}$  and the loaded  $Q_L$  factor have been obtained. Substitution of the (5)–(7) into (8) results in

$$\begin{aligned} S_{pp}(\omega) &= \frac{jQ_L S_{ppd} 2 \frac{(\omega - \omega_L)}{\omega_L} + \left\{ S_{ppd} + \frac{2R_c Y_{\text{ex1}}^2}{G_o (1 + \beta_1 + \beta_2)} \right\}}{jQ_L 2 \frac{(\omega - \omega_L)}{\omega_L} + 1} \end{aligned} \quad (19)$$

where  $S_{ppd}$  represents the detuned value of the reflection coefficient at the port  $p$  of 1 or 2 when the frequency is far from  $\omega_L$ . Similarly to  $S_{21}$ , expressed by (12), (19) is also of the fractional linear form

$$S_{pp} = \frac{a_1 t + a_2}{a_3 t + 1} \quad (20)$$

and, hence, it is also suitable for a fractional linear curve-fitting technique. The complex constants in (20) are related to important points on the  $Q$  circles: the ratio  $a_1/a_3$  represents the detuned value of the complex reflection coefficient  $S_{ppd}$ ,  $a_2$  is the reflection coefficient at the resonant frequency, and the imaginary part of  $a_3$  is equal to  $Q_L$ , namely,

$$\begin{aligned} a_1 &= jQ_L S_{ppd} \\ a_2 &= S_{ppd} + \frac{2R_c Y_{\text{ex1}}^2}{G_o (1 + \beta_1 + \beta_2)} \\ a_3 &= jQ_L. \end{aligned} \quad (21)$$

The complex constant  $a_3$  is the same as for the transmission case solution, which means that  $Q_L$  for transmission-mode resonators can be found either from the transmission or the reflection responses. The diameter of reflection circles can be obtained from the detuned and resonance points, which lie diametrically opposite to each other on the  $Q$  circle.

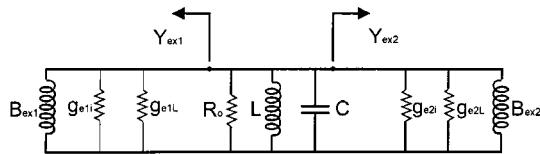


Fig. 11. Equivalent circuit of the transmission resonator to model coupling losses.

### B. Derivation of a Relationship Between Coupling Coefficients $\beta_1$ and $\beta_2$ and $S_{11}$ and $S_{22}$

Values of coupling coefficients of a transmission resonator system cannot be obtained directly from data sets of  $S_{11}$  and  $S_{22}$  measured near the resonant frequency. To solve the problem, we have employed a concept of splitting the coupling coefficients into two parts, i.e., lossless and lossy, applied before in [12] for the reflection-mode resonators. To model the coupling losses, we used the equivalent-circuit model shown in Fig. 11. The model of Fig. 11 is similar to the model a reflection mode (one-port) resonator in [4], but contains two ports. The coupling coefficient on each of the resonator ports can be expressed as

$$\beta_p = \frac{G_{\text{exp}}}{G_o} = \left( \frac{g_{epi}}{G_o} \right) + \left( \frac{g_{epL}}{G_o} \right) \quad (22)$$

where the external conductance  $G_{\text{exp}}$  is considered as the sum of the ideal lossless part  $g_{epi}$  and an additional lossy part  $g_{epL}$ . Hence, the total coupling coefficients can be considered as a sum of lossless  $\beta_i$  and lossy  $\beta_L$  terms, namely,

$$\beta_1 = \beta_{1i} + \beta_{1L} \text{ and } \beta_2 = \beta_{2i} + \beta_{2L} \quad (23)$$

where the lossless and lossy coupling coefficients are

$$\beta_{pi} = \frac{g_{epi}}{G_o} = \frac{R_o R_c}{(R_c + R_{sp})^2 + X_{sp}^2} \quad (24)$$

$$\beta_{pL} = \frac{g_{epL}}{G_o} = \frac{R_o R_{sp}}{(R_c + R_{sp})^2 + X_{sp}^2}. \quad (25)$$

For reflection-mode resonators, it has been shown [4] that the lossless and lossy parts of the coupling coefficient can be calculated from the diameter of the reflection  $Q$  circle and the diameter of the *coupling-loss* circle. We have obtained  $\beta_{pi}$  and  $\beta_{pL}$  terms for the transmission-mode resonators using the same geometrical approach, but solving for the four components of coupling coefficients instead of two.

Diameters of the  $Q$  circles have been determined understanding that a detuned point lies diametrically opposite a resonant point for any  $Q$  circle, i.e.,

$$\text{diameter of } Q \text{ circle} = \left| a_2 - \frac{a_1}{a_3} \right|.$$

Hence, the diameter of the transmission  $S_{21}$   $Q$  circle can be described as

$$\text{diameter of } S_{21} \text{ } Q \text{ circle} = \frac{2R_o R_c |Y_{\text{ex}1}| |Y_{\text{ex}2}|}{(1 + \beta_1 + \beta_2)} \quad (26)$$

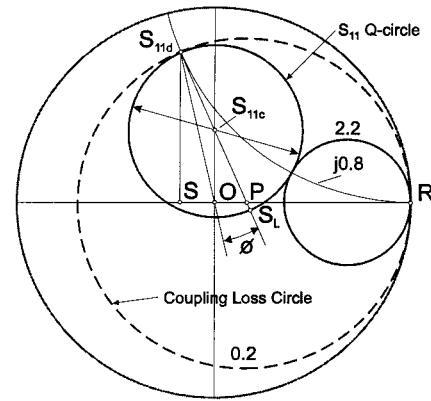


Fig. 12.  $S_{11}$   $Q$  circle and coupling loss circle based on [4].

and diameters of the reflection ( $S_{11}$  or  $S_{22}$ )  $Q$  circles are

$$\text{diameter of } S_{pp} \text{ } Q \text{ circle} = \left| \frac{2R_o R_c Y_{\text{exp}}^2}{(1 + \beta_1 + \beta_2)} \right| = \frac{2R_o R_c |Y_{\text{exp}}|^2}{(1 + \beta_1 + \beta_2)}. \quad (27)$$

By substituting (4) into (26) and (27) and knowing the form of the lossless part of the coupling coefficient described by (24), the diameters of the transmission and reflection  $Q$  circles have been expressed in terms of the coupling coefficients as

$$\text{diameter of } S_{21} \text{ } Q \text{ circle} = \frac{2\sqrt{\beta_{1i}}\sqrt{\beta_{2i}}}{1 + \beta_1 + \beta_2} \quad (28)$$

$$\text{diameter of } S_{11} \text{ } Q \text{ circle} = \frac{2\beta_{1i}}{1 + \beta_1 + \beta_2} \quad (29)$$

$$\text{diameter of } S_{22} \text{ } Q \text{ circle} = \frac{2\beta_{2i}}{1 + \beta_1 + \beta_2}. \quad (30)$$

The relationship of the coupling-loss circles to the coupling coefficients, following [4], is illustrated in Fig. 12 for port 1 of the resonator. For an ideal case, when observations are made at the real input of the resonator (point  $R$ ), the associated coupling loss circle can be visualized as a constant resistance circle that passes through the detuned reflection point  $S_{11d}$  of the  $S_{11}$   $Q$  circle and the point  $R = 1 + j0$  on the Smith chart. The diameter of the coupling loss circle can be found as

$$(\text{diameter of loss circle})|_{\text{port } p} = \frac{1 - |S_{ppd}|^2}{1 - |S_{ppd}| \cos \phi} \quad (31)$$

where  $\phi$  is the angle between the vectors  $\overrightarrow{S_{ppd}O}$  and  $\overrightarrow{S_{ppd}S_{ppc}}$ , where  $S_{ppd}$  is the detuned point and  $S_{ppc}$  represents the center of the respective  $Q$  circle.

To obtain the equations relating the diameters of the coupling-loss circles to the lossless and lossy parts of the coupling coefficients, one needs to consider that the equation for the diameter of the coupling-loss circle at port  $p$  is

$$d_p = \frac{2}{1 + \frac{R_{sp}}{R_c}} \quad (32)$$

and that based on (25) and (26)

$$\frac{R_{sp}}{R_c} = \frac{g_{epL}}{g_{epi}} = \frac{\beta_{pL}}{\beta_{pi}}. \quad (33)$$

Hence, the diameter of the coupling loss circle can be obtained as

$$d_p = \frac{2}{1 + \frac{\beta_{pL}}{\beta_{pi}}}. \quad (34)$$

Solving (29), (30) and (34) for coupling coefficients  $\beta_{1i}$ ,  $\beta_{1L}$ ,  $\beta_{2i}$ , and  $\beta_{2L}$  yield the following expressions:

$$\begin{aligned} \beta_{1i} &= \frac{x}{2 \left[ 1 - \left( \frac{x}{d_1} + \frac{y}{d_2} \right) \right]} \\ \beta_{1L} &= \left( \frac{2}{d_1} - 1 \right) \beta_{1i} \\ \beta_{2i} &= \frac{y}{2 \left[ 1 - \left( \frac{x}{d_1} + \frac{y}{d_2} \right) \right]} \\ \beta_{2L} &= \left( \frac{2}{d_2} - 1 \right) \beta_{2i} \end{aligned} \quad (35)$$

where  $d_1$  is a diameter of the port 1 coupling-loss circle,  $d_2$  is a diameter of the port 2 coupling-loss circle,  $x$  is a diameter of the port 1 ( $S_{11}$ )  $Q$  circle, and  $y$  is a diameter of the port 2 ( $S_{22}$ )  $Q$  circle.

Using the equations derived in this paper, the values of the coupling coefficients and the loaded  $Q_L$  factor of a transmission-mode dielectric resonator can be obtained from data sets of  $S_{11}$ ,  $S_{22}$ , and  $S_{21}$  measured around the resonance and by applying curve-fitting procedures.

#### IV. ASSESSMENT OF THE ACCURACY OF THE TRANSMISSION-MODE $Q_o$ -FACTOR TECHNIQUE USING COMPUTER SIMULATIONS

To assess the accuracy of the presented  $Q$ -factor measurement technique, several computer simulations have been performed. The transmission-mode  $Q$ -factor (TMQF) technique was first applied to ideal  $Q$ -circle data sets. Applying the  $Q_{Fit}$  software implementation of the developed technique to the ideal  $S$ -parameter data sets yielded perfect recovery of the unloaded  $Q$  factor. The TMQF technique was next applied to  $Q$  circles influenced by noise, electrical delay of transmission lines, coupling losses, and coupling reactances.  $Q$ -circle test sets were generated using the software  $Q_{Gen}$  [16]. By applying the TMQF procedure to each  $Q$ -circle test set, the obtained values of unloaded  $Q_o$  factor, loaded  $Q_L$  factor, and coupling coefficients results were compared to the nominal values for the test. Simulations were done for  $Q$  factors of 9000 and

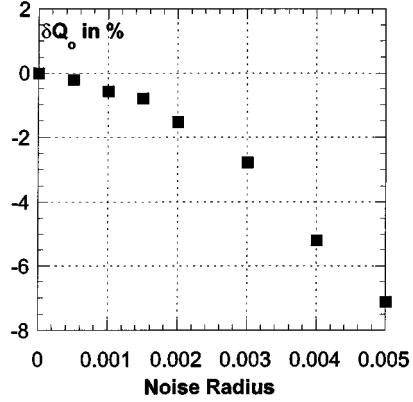


Fig. 13. Calculated error in  $Q_o$  from  $Q_L$  of  $S_{21}$  fit versus noise radius for  $Q_o$  of 9000.

10 000, similar to values of sapphire dielectric resonators used for verification of the TMQF technique in Section V.

##### A. Investigation of Noise Influence on the Unloaded $Q_o$ Factor Obtained From the TMQF Technique

The effect of noise on the TMQF technique was investigated over a range of noise levels from 0.0005 to 0.005, equivalent to a SNR from 45 to 23 dB. The simulations were performed for the worst-case condition where the same noise was added to all three parameters  $S_{21}$ ,  $S_{11}$ , and  $S_{22}$  for each noise level tested. To set the location of the  $Q$  circles on the complex plane in positions typical for a practical case, the following parameters of the circuit model of the resonator system of Fig. 10 were used:

- 1) coupling-loss resistances  $R_{s1} = R_{s2} = 5 \Omega$ ;
- 2) coupling reactances  $X_{s1} = X_{s2} = 20 \Omega$ ;
- 3) shunt resistance of the ideal resonator  $R_o = 5 \Omega$ ;
- 4) source resistance  $R_c = 50 \Omega$ ;
- 5) transmission line to wavelength ratio  $L/\lambda = 0$ ;
- 6) unloaded  $Q_o$  factor = 9000,  $f_o = 10$  GHz;
- 7) coupling coefficients  $\beta_1 = \beta_2 = 0.080$ ,  $N = 401$  points;
- 8) noise radius ranging from 0.0005 to 0.005.

In the simulation, equal couplings on both ports were used and the frequency span of all  $Q$  circles was chosen to be six times the loaded  $Q$  bandwidth centered about the resonant frequency. Simulated Gaussian noise data sets were generated using MATLAB V5.3 [19] and the random noise was superimposed on noise-free  $S$ -parameter data. Ten independent simulations were performed for each noise level investigated. The  $Q_{Fit}$  program was applied to each of the  $Q$ -circle test sets ( $S_{21}$ ,  $S_{11}$ , and  $S_{22}$ ) to calculate the loaded  $Q_L$  factor, the coupling coefficients, and the unloaded  $Q_o$  factor. The difference between the ideal and calculated  $Q$ -factor values is shown in Fig. 13. The simulation results show that the transmission-mode  $Q_o$  factor technique can be used to obtain the unloaded  $Q_o$  factor from relatively noisy synthesized  $S$ -parameter trace data (up to a noise radius of 0.004) with error not exceeding 5%. In many practical measurements performed by the authors, the noise radius did not exceed 0.001. For this value of noise radius, the simulated error of the TMQF technique is better than 0.6%.

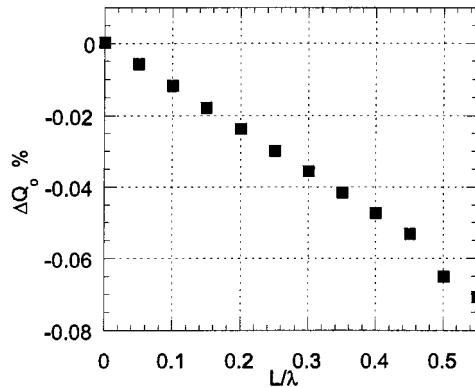


Fig. 14. Calculated error in  $Q_o$  using  $Q_L$  of the  $S_{21}$  fit for  $Q_o$  of 10 000.

### B. Investigation of Cable Influence on the Unloaded $Q_o$ Factor Obtained From the TMQF Technique

Errors in the calculations of the unloaded  $Q_o$  factor due to delay introduced by transmission lines when using the TMQF technique were investigated for short and long transmission lines. For investigations of the effect of delay, the following parameters were selected to simulate the equal coupling condition ( $\beta_1 = \beta_2 = 0.08$ ) and to place the  $Q$  circles in positions typical for practical measurements on the complex plane:

- 1)  $R_{s1} = R_{s2} = 5 \Omega$ ;
- 2)  $R_{s1} = R_{s2} = 5 \Omega$ ;
- 3)  $X_{s1} = X_{s2} = 20 \Omega$ ;
- 4)  $R_o = 5 \Omega$ ;
- 5)  $R_c = 50 \Omega$ ;
- 6)  $N = 401$  points.

The simulations were performed for short and long cable lengths  $L$ . For the short cables, the analysis was done for the  $L/\lambda$  ratio varying from 0 to  $0.55\lambda$ ; for the long cable, the analysis was for one case, namely,  $L/\lambda = 30$ .

Errors in the  $Q_o$  factor calculated using the TMQF technique for transmission-line length  $L/\lambda$  from 0 to 0.55 are shown in Fig. 14. The error in the calculated  $Q_o$  value increases from 0% to 0.07% with the increase of the line length. The difference in the set and calculated values of the  $Q_o$  factors result mostly from errors in the coupling coefficients' calculations, as errors in the  $Q_L$ -factor values obtained from the  $S_{21}$  fit are below 0.000 03%.

Simulations of the TMQF technique with cables 30 wavelengths long connected on each side of the resonator resulted in the fitted  $Q_o$  of 9677, as compared to the actual value of 10 000. The error in for this case was  $-3.23\%$ .

## V. MEASUREMENTS OF THE $Q$ FACTOR OF DIELECTRIC RESONATORS USING THE TRANSMISSION-MODE $Q_o$ -FACTORIZATION TECHNIQUE

To verify applicability of the TMQF technique under differing measurement conditions, the technique has been used for measurements of dielectric resonators of differing values of the  $Q$  factor. The measurements were performed on a 10-GHz sapphire resonator with a copper cavity and copper endplates at room temperature of the  $Q_o$  factor of 8400, 10-GHz sapphire

resonator with a copper cavity and superconducting endplates at cryogenic temperatures with  $Q_o$  factors ranging from 8400 to 1 000 000; 25-GHz sapphire resonator with a silver-plated cavity and superconducting endplates with a  $Q_o$  factor of 6000 to 300 000, a 17-GHz rexolite resonator with a copper cavity and superconducting endplates of the  $Q_o$  factor ranging from 4500 to 10 000; 19-GHz Teflon resonator with copper cavity and superconductor endplates at a cryogenic temperature of the  $Q_o$  factor of 63 000–190 000; 17.6-GHz polyethylene resonator with copper cavity and superconducting endplates at a cryogenic temperature of the  $Q_o$  factor of 10 000–20 000; 18-GHz strontium–lanthanum–aluminate resonator at room temperature with copper cavity and copper endplates of the  $Q_o$  factor of 4560. Measurements of the surface resistance of high-temperature superconductors (HTSs) in a 25-GHz sapphire resonator of the  $Q$  factor of approximately 420 000 using the TMQF technique were presented in [16] and [17].

For the verification of the TMQF technique in this paper, we have chosen a resonator with the  $Q$  factor of approximately 8400. Measurements of a small  $Q$  factor are more difficult than of a large  $Q$ -factor value, hence, the verification performed for the small value allows to draw more general conclusions. Measured data and subsequent processing steps using the TMQF technique for the 10-GHz sapphire resonator in a copper cavity at room temperature are presented in Section V-A. The verification measurement system consisted of a chosen resonator, an HP 8722C Vector Network Analyzer, APD DE204SL closed-cycle cryocooler system, two 60-cm low-loss semirigid cables (each cable terminated with a coupling loop), Neocera LTC-10 temperature controller fitted with a 50-W resistive heating element, two silicon diodes temperature sensors, and an IBM-PC computer.

### A. Verification of the TMQF Technique for the Resonator With the $Q_o$ Factor of 8400

The resonator chosen for verification contained a sapphire rod of 7.41-mm height and 12.3 2-mm diameter enclosed in the copper cavity of 24-mm diameter [18]. The geometrical factors of the resonator were  $A_m = 27617$ ,  $A_s = 279$ ,  $p_e = 0.971$ , and  $\epsilon_r = 9.38$ . The sapphire rod was assumed to have  $\tan \delta$  of  $5 \times 10^{-8}$ . The measurements were performed with microwave source power of  $-15$  dBm.

The 10-GHz sapphire resonator with moderate coupling had the insertion loss at a resonance of  $-20.8$  dB. Between 579–1601 measurement points around the resonance were logged for each of the scattering parameters  $S_{21}$ ,  $S_{11}$ , and  $S_{22}$ . Figs. 15 and 16 present raw measurement data and Figs. 17–20 give step-by-step correction of data using the transmission-mode  $Q_o$ -factor technique.

In Fig. 15, 1601 points representing  $S_{21}$  values measured with the vector network analyzer, as well as the fitted circle obtained using the basic TMQF technique are shown. The curve fit implemented in the  $Q_{Fit}$  software of the TMQF technique [16] resulted in a loaded  $Q_L$  factor of 7393 and a resonant frequency of 9.924949 GHz. The SNR of the trace was high (46.7 dB) and the trace was not “contaminated” by the frequency dependence

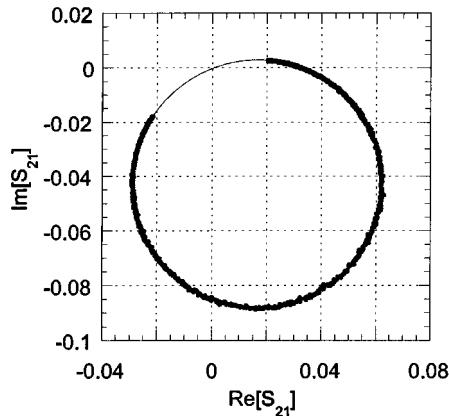


Fig. 15. Measured  $S_{21}$  data with 1601 points spanning 5 MHz and a fitted  $S_{21}$   $Q$  circle.

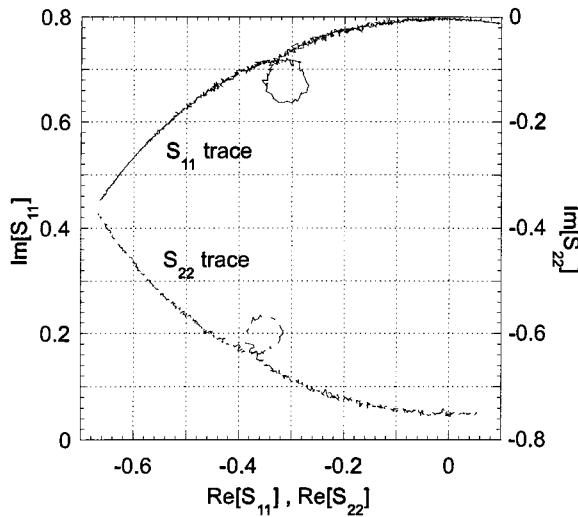


Fig. 16. Measured  $S_{11}$  data and  $S_{22}$  data including off-resonance regions.

of delay; hence, no further processing of  $S_{21}$  was necessary. The SNR was calculated as follows:

$$\text{SNR}_{dB} = 20 \log_{10} \frac{R}{\sqrt{\frac{1}{N} \sum_{i=1}^N (R - r_i)^2}} \quad (36)$$

where  $R$  is the radius of the  $Q$  circle and  $r_i$  is the magnitude of the  $i$ th point of the  $N$  point  $S$ -parameter data set obtained from measurements around the resonance.

Fig. 16(a) and (b) shows measured reflection  $S_{11}$  and  $S_{22}$  traces containing 710 and 579 points, respectively. Wide measurement spans of 44.3 and 36.1 MHz were used to obtain the resonant circles, as well as significant parts of the off resonance circles so that the correction of the impedance mismatch and the phase distortion of the  $S_{11}$ - and  $S_{22}$ -parameters could be performed. To improve resolution and SNR in the resonant circles, narrow span measurements would be better. However, it would have required taking measurements of  $S_{11}$  and  $S_{22}$  data twice. Hence, it is more economical to take  $S_{11}$  and  $S_{22}$  measurements in wide span as noise is removed from the data through the

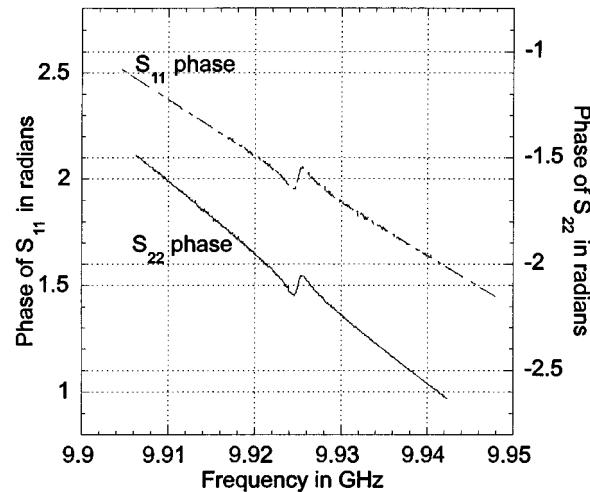
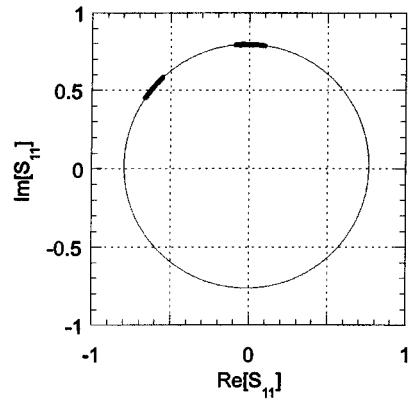
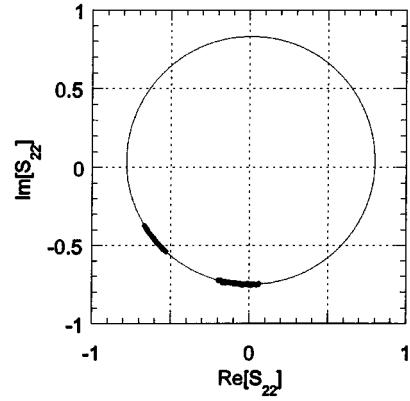


Fig. 17. Phase of  $S_{11}$  and of  $S_{22}$  versus frequency over a wide bandwith.



(a)



(b)

Fig. 18. Reconstructed off resonance circles of: (a)  $S_{11}$ - and (b)  $S_{22}$ -parameters.

curve-fitting procedure. The SNR of  $S_{11}$  and  $S_{22}$  traces were 22.9 and 23.6 dB, respectively.

The reflection traces in Fig. 16 are clearly distorted due to impedance mismatch resulting in shift of the circles from their ideal locations. Center coordinates for the  $S_{11}$  circle are 0.01593 and 0.01669 with a radius of 1.559. For the  $S_{22}$  circle, coordinates of the center are 0.00776 and 0.03976 with a radius of 1.5812. Phase distortion is also present due to uncalibrated cables, and this is visible in Fig. 17, presenting the phase of  $S_{11}$

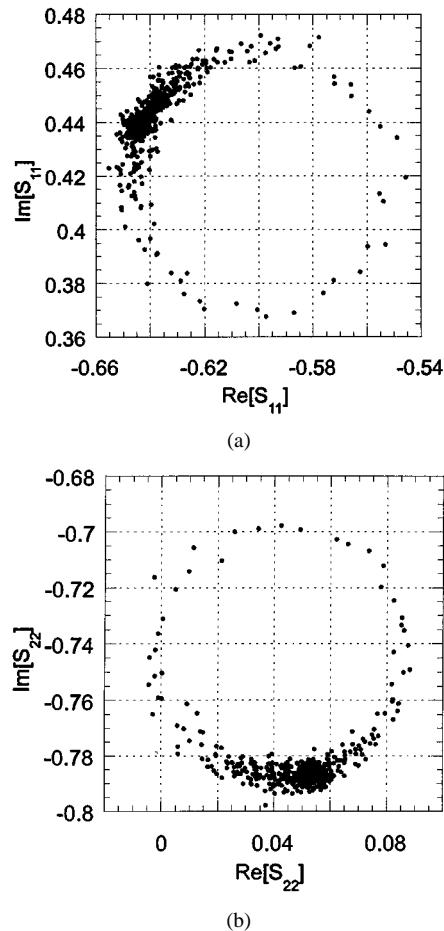


Fig. 19. Phase corrected: (a)  $S_{11}$   $Q$  circle with 710 points and a span of 44.3 MHz and (b)  $S_{22}$   $Q$  circle with 579 points and a span of 36.1 MHz.

and  $S_{22}$  versus frequency. Both distortions need to be removed from the measurement data before coupling coefficients may be calculated.

The linear fits of the wide span  $S_{11}$  and  $S_{22}$  phase data of Fig. 17 give the rate of change in phase of  $-2.459 \times 10^{-8}$  rad/Hz and  $-3.161 \times 10^{-8}$  rad/Hz, respectively. These values of the phase rate were used to correct the  $S_{11}$  and  $S_{22}$  resonance circles. The off-resonance traces of Fig. 16 allowed for the reconstruction of the off resonance circles and for compensation for the offset, as shown in Fig. 18.

The phase and mismatch corrected resonant  $S_{11}$  and  $S_{22}$  circles are shown in Fig. 19. As can be seen the corrected reflection, resonant circles contain a large concentration of points around the detuned regions, which would cause a heavy bias in the fitting procedure. To remove the influence of the detuned regions, the end sections of the  $S_{11}$  and  $S_{22}$   $Q$  circles are deleted.

Fig. 20 shows reduced data sets of the corrected  $S_{11}$  (71 points, span = 4.375 MHz) and  $S_{22}$   $Q$  circles (66 points, span = 4.063 MHz) and the resulting fitted circles. Using the procedure described in Section III-B and the results shown in Fig. 20, the coupling coefficient  $\beta_1$  and  $\beta_2$  of 0.061 and 0.056, respectively, were obtained. The lossless and lossy parts of  $\beta_1$  and  $\beta_2$  were equal to 0.0544 and 0.0069 and 0.04996 and 0.0059, respectively.

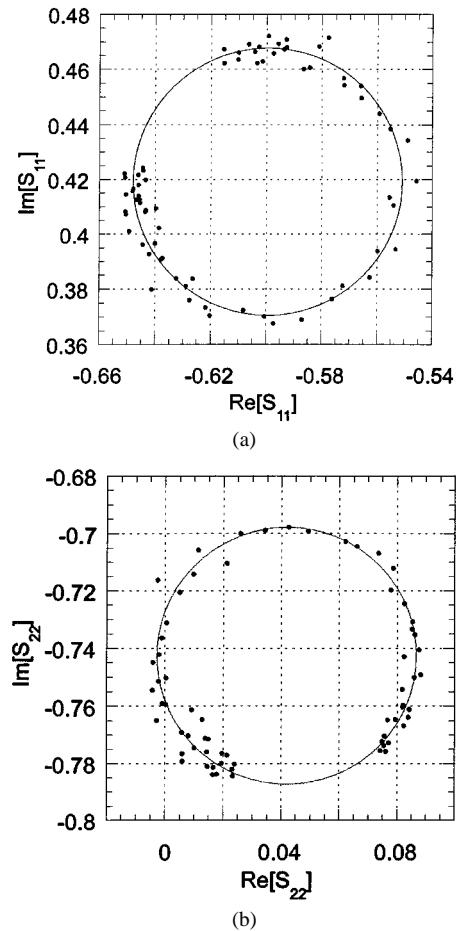


Fig. 20. Pruned phase corrected: (a)  $S_{11}$   $Q$  circle and (b)  $S_{22}$  circle.

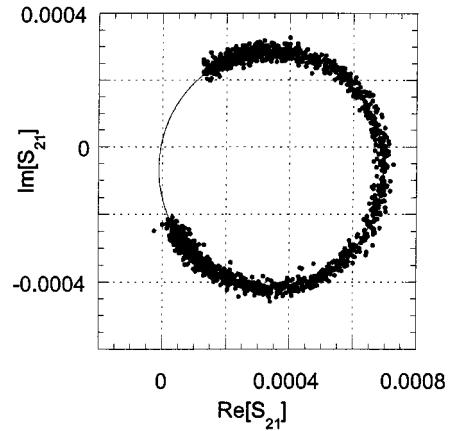


Fig. 21. Measured 1601 points  $S_{21}$  and fitted  $S_{21}$   $Q$  circle for the case of very weak coupling with a 3-MHz span.

Using the computed values of the coupling coefficients and loaded  $Q$  factor, the unloaded  $Q$  factor  $Q_o$  of 8258.3 was obtained.

The same 10-GHz resonator was measured with a very weak coupling since the  $Q_o$  factor can be approximated by the loaded  $Q_L$  factor ( $Q_o \approx Q_L|_{\beta_1 \approx \beta_2 \approx 0}$ ). The measured  $S_{21}$  trace is shown in Fig. 21 with 1601 points spanning 3 MHz. The resulting insertion loss at resonance was  $-63$  dB and the SNR ratio for the  $S_{21}$  trace was 28.2 dB. Under this weak coupling, the  $S_{11}$  and  $S_{22}$  reflection  $Q$  circles were too small to be seen

TABLE II  
 $Q$ -FACTORS OF VARIOUS RESONATORS MEASURED USING THE TMQF TECHNIQUE

Frequency in GHz	Dielectric	Endwall Material	Qo-Factor
10	sapphire	superconductor	8,400 - 1,000,000
17	rexolite	superconductor	4,500 - 10,000
17.6	polyethylene	superconductor	10,000 - 20,000
18	strontium lanthanum aluminate	copper	4,560
19	teflon	superconductor	63,000 - 190,000
25	sapphire	superconductor	420,000

even with the highest resolution. The data scatter  $D_s$  for the  $S_{21}$  trace was calculated to be 0.072, which did not exceed the recommended limit of 0.1 for reliable results. Hence, the circle-fitting procedure provides reliable fit results without any further processing.

Using the TMQF fitting procedure, the loaded  $Q_L$  factor (and, hence, the  $Q_o$  factor) of 8382 was obtained. Fitted resonant frequency was 9.924136 GHz. The difference between the unloaded  $Q_o$  factor obtained from the measurements under very weak coupling ( $\text{IL} = -63$  dB) and the moderate coupling ( $\text{IL} = -20.8$  dB) was only  $-1.5\%$ . This good agreement of the  $Q_o$ -factor values for two very different couplings shows the high accuracy of the developed TMQF technique for unloaded  $Q_o$ -factor measurements under the influence of noise, transmission line delay, and coupling losses and reactances.

Apart from measurements presented above, we performed tests of dielectric resonators with various  $Q$ -factor values, as listed in Table II.

## VI. CONCLUSIONS

The presented transmission-mode  $Q$ -factor technique has proven to be a reliable tool for accurate measurements of the unloaded  $Q_o$  factor of microwave resonators in the presence of noise, uncalibrated cables and connectors, impedance mismatch, crosstalk, and coupling loss. Computer simulations have shown that the addition of noise with a noise radius below 0.001 results in an error smaller than 0.6% in calculated values of the  $Q_o$  factor for the nominal value of 9000. Simulations with uncalibrated cables added resulted in errors of 0.07% for two cables 0.55 wavelengths long and  $-3.3\%$  for two cables 30 wavelengths long. The error in the  $Q_o$ -factor calculations due to the uncalibrated cables using the TMQF technique decreases as the  $Q$  increases by the same factor.

Measurements of the 10-GHz dielectric resonator with the  $Q_o$  factor of the order of 9000 at a very low coupling and at a moderate coupling resulted in a difference of only 1.5%. We have applied the developed method to several  $Q_o$  measurements of sapphire, rexolite, strontium-lanthanum-aluminate ( $\text{SrLaAlO}_4$ ) (SLA), Teflon, and polyethylene resonators in the range from 4560 to 1 000 000 at cryogenic temperatures and have found the performance very satisfactory. In general, we conclude that the TMQF technique can be applied for the  $Q$ -factor measurements above 1000.

## ACKNOWLEDGMENT

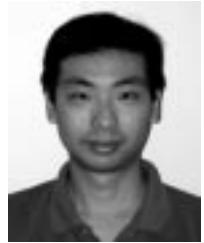
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